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SOBOLEV SINFIDA OPERATOR QATNASHGAN TENGLAMALAR UCHUN BOSHLANG‘ICH-CHEGARAVIY MASALALAR

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Annotatsiya. *To‘lqin tenglamasining umumlashgan yechimi va shu tenglamaga mos Koshi masalasi, to‘lqin tenglamasi uchun boshlang‘ich-chegaraviy masalaning Sobolev sinfidagi yechimi tadqiq etildi va yangi natijalar olindi.*

Boshlang‘ich-chegaraviy masalaning umumlashgan yechimining silliqligi va shuning natijasida klassik yechiming mavjudligi va yagonaligi isbot qilindi, hamda natijalar bir nechta teoremalar shaklida keltirildi.

Kalit so’zlar: *Boshlang‘ich-chegaraviy masala, elliptik operatorlar qatnash-gan masalaning turlari, elliptik tipdagi masalalar, umumlashgan yechimning mavjudligi, Sobolev fazosi, umumlashgan yechimning silliqligi.*

$H^l(G)$ **Sobolev fazosi.** Avval biz $\bar{G}$ yopiq chegaralangan sohada l marta uzluksiz differensiallanuvchi bo‘lgan $u(x,y,z)$ funksiyalarning

$$\|u\| = \left\{ \sum_{0\le|\alpha|\le l} \iiint_{\bar{G}} \left(D^\alpha u(x,y,z)\right)^2 dxdydz \right\}^{\frac{1}{2}}$$

norma bilan kiritilgan $\tilde{H}^l(\bar{G})$ normallangan fazosini qaraymiz.

Bu norma bo‘yicha $\tilde{H}^l(\bar{G})$ normallangan fazoning to‘ldirilmasi $H^l(G)$ orqali belgilanadi. $\{u_n(x,y,z)\} \in \tilde{H}^l(\bar{G})$ ketma–ketlik $\tilde{H}^l(\bar{G})$ fazoda fundamental ketma–ketlik bo‘lsin, ya’ni $n,m\to\infty$ da $\|u_n - u_m\|_{\tilde{H}^l(\bar{G})} \to 0$ bo‘lsin. Ma’lumki, $\tilde{H}^l(\bar{G})$ fazodagi norma

$$\|u\|^2_{\tilde{H}^l(\bar{G})} = \sum_{0\le|\alpha|\le l} \|D^\alpha u\|^2_{\tilde{L}_2(\bar{G})}$$

shaklida bo‘lganligi uchun $|\alpha| = 0, 1, \ldots, l$ bo‘lgan har bir α multiindeks uchun $\{D^\alpha u_n\}$ ketma–ketlik $\tilde{L}_2(\bar{G})$ fazoda fundamental ketma–ketlikdan iborat bo‘ladi.

$L_2(G)$ fazoning to‘la ekanligidan bu fazoda biz $D^\alpha u$, $0 \le |\alpha| \le l$ orqali belgilaydigan elementlar mavjud bo‘lib, bunda $n\to\infty$ da o‘rtacha ma’noda $D^\alpha u_n \to D^\alpha u$ yaqinlashuvchi bo‘ladi. Agar $\alpha \ne 0$ bo‘lsa, u holda $D^\alpha u$ elementga *umumlashgan xususiy hosila* deb aytiladi. $\alpha = (\alpha_1, \alpha_2, \alpha_3)$ bo‘lsin. U holda

$$D^{\alpha}u = \frac{\partial^{|\alpha|}u}{\partial x^{\alpha_1}\partial y^{\alpha_2}\partial z^{\alpha_3}}, \qquad |\alpha| = (\alpha_1, \alpha_2, \alpha_3)$$

bo‘ladi. Bu $H^{l}(G)$ fazo

$$(u,v)_{H^{l}(G)} = \sum_{0\le|\alpha\le l|}\left(D^{\alpha}u, D^{\alpha}v\right)_{L_2(G)}$$

skalyar ko‘paytmaga nisbatan Gilbert fazosidan iborat bo‘ladi.

S.L. Sobolevning joylashish haqidagi quyidagi teoremasi o‘rinlidir.

1–teorema (S.L. Sobolev teoremasi). *$G \subset R^3$ bir bog‘lamli chegara-langan soha bo‘lib, uning chegarasi ∂G esa l marta uzluksiz differensiallanuvchi bo‘lsin, bunda $l \ge 2$. U holda $H^{l}(G)$ fazo $C^{l-2}(\overline{G})$ fazoda joylashgan bo‘ladi.*

Har bir $s \in N$ son

$$H^{s}(\Omega) = \left\{ f \in L_2(\Omega),\ D^{\alpha} f \in L_2(\Omega),\ |\alpha| \le s \right\}$$

tenglik bilan aniqlangan fazo bo‘ladi, bunda $D^{\alpha} f$ taqsimot bo‘lib, f taqsimotni differensiallash natijasida hosil qilingan. Bu fazoda f va g elementlarning skalyar ko‘paytmasi

$$(f,g)_s = \int_{\Omega}\sum_{|a|\le s} D^{\alpha} f(x)\overline{D^{\alpha} g(x)}dx$$

formula orqali aniqlanadi.

Umumlashgan funksiyalarning silliqligi. Umumlashgan yechimning silliqligini o‘rganilishida birinchi va ikkinchi ((2.2.5) $\sigma \equiv 0$ chegaraviy shartlarda) aralash masalalar uchun (2.2.1) to‘lqin tenglamasining ((2.2.1) da $k \equiv 1$, $a \equiv 0$) xususiy tenglama holatini qarash bilan cheklanamiz, tenglama va σ funksiyalarining koeffitsientlarining silliqligi ham umumiy holatda xuddi shunday natijalar ushbu usul bilan o‘rnatiladi. [10]

Aytaylik, $u(x,t)$ - to‘lqin tenglamasi uchun birinchi va ikkinchi aralash masalaning umumlashgan yechimi bo‘lsin

$$u_{tt} - \Delta u = f(x,t), \tag{1}$$

$$u\big|_{t=0} = \varphi(x), \quad u_t\big|_{t=0} = \psi(x), \tag{2}$$

va yoki ikkinchi aralash masalada

$$u\big|_{\Gamma_t} = 0. \tag{3}$$

(1)–(3) va (1), (2), (4) masalalari umumlashgan (yagona) yechimga ega, agar $\psi \in L_2(D)$, $f \in L_2(Q_T)$ bo‘lsa, φ funksiya bo‘lsa, birinchi masalada $\overset{0}{H}{}^1(D_t)$ fazoga yoki ikkinchi masalada $H^1(D)$ fazoga tegishli. Bundan $u(x,t)$ umumlashgan yechimlarning har biri $H^1(Q_T)$ da quyidagi qatorga keltiriladi

$$u(x,t) = \sum_{k=1}^{\infty} U_k(t) v_k(x) \tag{5}$$

bunda

$$U_k(t) = \varphi_k \cos\sqrt{-\lambda_k}\,t + \frac{\psi_k}{\sqrt{-\lambda_k}} \sin\sqrt{-\lambda_k}\,t +$$

$$+\frac{1}{\sqrt{-\lambda_k}} \int_0^t f_k(\tau) \sin\sqrt{-\lambda_k}\,(t-\tau)\,d\tau, \qquad k = 1,2,\ldots \tag{6}$$

(ikkinchi aralash masalada

$$U_1(t) = \varphi_1 + t\psi_1 + \int_0^t (t-\tau) f_1(\tau)\,d\tau =$$

$$\lim_{\lambda\to 0}\left(\varphi_1 \cos\sqrt{-\lambda}\,t + \frac{\psi_1}{\sqrt{-\lambda}} \sin\sqrt{-\lambda}\,t + \frac{1}{\sqrt{-\lambda}} \int_0^t f_1(\tau) \sin\sqrt{-\lambda}\,(t-\tau) \right)),$$

$$\varphi_k = (\varphi, v_k)_{L_2(D)}, \quad \psi_k = (\psi, v_k)_{L_2(D)},$$

$$f_k(t) = \int_{D_t} f(x,t) v_k(x)\,dx, \qquad k = 1,2,\ldots., \tag{7}$$

$v_1, v_2, \ldots$ va $\lambda_1, \lambda_2, \ldots$ lar umumlashgan o‘ziga funksiyalar va mos o‘ziga xos birinchi ((1) – (3) masalalarga qaralsa) yoki ikkinchi ((1), (2), (4) masalalarga qaralsa) qiymatlar ketma-ketligi D da Laplas operatori uchun chegaraviy masala (birinchi chegaraviy masalada barcha $k = 1,2,\ldots$ larda $\lambda_k < 0$, ikkinchi chegaraviy masalalarda $k = 2,3,\ldots$ da $\lambda_k < 0$ va $\lambda_1 = 0$, chunki, $v_1 = const = 1/\sqrt{|D|}$).

Aytaylik, ba’zi $s \geq 1$ da D sohasining ∂D chegarasi C^s sinfiga tegishli. U holda, Laplas operator uchun birinchi va ikkinchi chegaraviy masalaning $v_k(x)$, $k = 1,2,\ldots$ o‘ziga xos funksiyalari mos $H_D^s(D)$ va $H_N^s(D)$ fazolariga tegishli, ya’ni $H^s(D)$ ga tegishli va ∂D da birinchi chegaraviy masala uchun quyidagi chegaraviy shartlarni qanoatlantiradi

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$$v_k\big|_{\partial D} = \ldots = \Delta^{\left[\frac{s-1}{2}\right]} v_k\big|_{\partial D} = 0, \quad k = 1, 2, \ldots,$$

ikkinchi chegaraviy masalada $s > 1$ da quyidagi chegaraviy shartlar bilan

$$\frac{\partial v_k}{\partial n}\bigg|_{\partial D} = \ldots = \frac{\partial}{\partial n}\Delta^{\left[\frac{s-1}{2}\right]} v_k\big|_{\partial D} = 0, \quad k = 1, 2, \ldots,$$

Eslatib o‘tamiz $H_N^1(D) = H^1(D)$.

Aytaylik, (1)-(3) birinchi aralash masalalarda $\varphi \in H_D^s(D)$, $\psi \in H_D^{s-1}(D)$, f bo‘lsa $H^{s-1}(Q_T)$ fazoning $H_D^{s-1}(Q_T)$ qismfazosiga tegishli $s > 1$ da barcha $f \in H^{s-1}(Q_T)$ funksiyalardan tashkil topgan, ba’zi

$$f\big|_{\Gamma_T} = \ldots = \Delta^{\left[\frac{s}{2}\right]-1} f\big|_{\Gamma_T} = 0.$$

Bunda $s = 1$, $\tilde{H}_D^{s-1}(Q_T) = \tilde{H}_D^0(Q_T) = L_2(Q_T)$.

(1), (2), (4) ikkinchi aralash masalada $\varphi \in H_N^s(D)$, $\psi \in H_N^{s-1}(D)$ deb, f bo‘lsa $H^{s-1}(Q_T)$ fazoning $\tilde{H}_N^{s-1}(Q_T)$ qismfazosiga tegishli $s > 2$ da barcha $f \in H^{s-1}(Q_T)$ funksiyalardan tashkil topgan, ba’zi

$$\frac{\partial f}{\partial n}\bigg|_{\Gamma_T} = \ldots = \frac{\partial}{\partial n}\Delta^{\left[\frac{s-1}{2}\right]-1} f\bigg|_{\Gamma_T} = 0.$$

bunda $s = 2$ da $\tilde{H}_N^{s-1}(Q_T) = \tilde{H}_N^1(Q_T) = H^1(Q_T)$, $s = 1$ da $\tilde{H}_N^{s-1}(Q_T) = \tilde{H}_N^0(Q_T) = L_2(Q_T)$.

2-teorema. *Aytaylik, ba’zi* $s \geq 1$ *da* $\partial D \in C^s$ *va (1)-(3) birinchi aralash masalalarda* $\varphi \in H_D^s(D)$, $\psi \in H_D^{s-1}(D)$, $f \in \tilde{H}_D^{s-1}(Q_T)$, *(1), (2), (4) ikkinchi aralash masalalarda bo‘lsa* $\varphi \in H_N^s(D)$, $\psi \in H_N^{s-1}(D)$, $f \in \tilde{H}_N^{s-1}(Q_T)$ *bo‘lsin. U holda (5) qator* $H^s(D_t)$ *da* $u(x,t)$ *umumlashgan yechimga* $t \in [0,T]$ *bo‘yicha tegis yaqinlashadi. Bundan tashqari, ixtiyoriy* $p = 1,\ldots,s$ *uchun (5) qatordan* t *bo‘yicha* p*-marta hadlarida differensiallanuvchi,* $H^{s-p}(D_t)$ *da* $t \in [0,T]$ *bo‘yicha tegis yaqinlashuvchi qator hosil bo‘ladi va barcha* $t \in [0,T]$ *uchun quyidagi tenglik o‘rinli*

$$\sum_{p=0}^{s}\left\|\sum_{k=1}^{\infty}\frac{\partial^p}{\partial t^p}(U_k(t)v_k(t))\right\|^2_{H^{s-p}(D_t)}\le$$

$$\le C\left(\|\varphi\|^2_{H^s(D)}+\|\psi\|^2_{H^{s-1}(D)}+\|f\|_{H^{s-1}(Q_T)}\right). \qquad (8)$$

1-xulosa. *Aytaylik ba'zi* $s\ge 1$ *da* $\partial D\in C^s$ *va (1)-(3) birinchi aralash masalalarda* $\varphi\in H_D^s(D)$, $\psi\in H_D^{s-1}(D)$, $f\in\tilde{H}_D^{s-1}(Q_T)$, *(1), (2), (4) ikkinchi aralash masalalarda bo'lsa* $\varphi\in H_N^s(D)$, $\psi\in H_N^{s-1}(D)$, $f\in\tilde{H}_N^{s-1}(Q_T)$. *U holda bu masalalardan har birining umumlashgan yechimi* $H^s(Q_T)$ *ga tegishli va (5) qator* $H^s(Q_T)$ *da unga yaqinlashadi.*

1-lemma. *Agar* $f\in H^q(Q_T)$, $q\ge 0$ *va* $g\in L_2(D)$ *bo'lsa, u holda funksiya*

$$h(t)=\int_{D_t} f(x,t)g(x)dx$$

$H^q(0,T)$ *ga tegishli va quyidagi tenglik o'rinli*

$$\frac{\partial^p h(t)}{\partial t^p}=\int_{D_t}\frac{\partial^p f(x,t)}{\partial t^p}g(x)dx, \qquad 0\le p\le q\,.$$

2-lemma. *Agar ba'zi* $s\ge 1$ *da (1)-(3) birinchi aralash masalalarda* $\partial D\in C^s$ *va* $\varphi\in H_D^s(D)$, $\psi\in H_D^{s-1}(D)$, $f\in\tilde{H}_D^{s-1}(Q_T)$ *yoki (1), (2), (4) ikkinchi aralash masalalarda* $\varphi\in H_N^s(D)$, $\psi\in H_N^{s-1}(D)$, $f\in\tilde{H}_N^{s-1}(Q_T)$ *bo'lsa, u holda ixtiyoriy* $p\le s$ *da* $\sum_{k=1}^{\infty}\left(\frac{\partial^p U_k(t)}{\partial t^p}\right)^2|\lambda_k|^{s-p}$ *qator* $t\in[0,T]$ *bo'yicha tekis yaqinlashuvchi, va*

$$\sum_{k=1}^{\infty}\left(\frac{\partial^p U_k(t)}{\partial t^p}\right)^2|\lambda_k|^{s-p}\le C\left(\|\varphi\|^2_{H^s(D)}+\|\psi\|^2_{H^{s-1}(D)}+\|f\|_{H^{s-1}(Q_T)}\right), \qquad (9)$$

bunda $C>0$ *faqat* Q_T *ga bog'liq o'zgarmas.*

2-xulosa. *Aytaylik,* $\partial D\in C^2$ *va* $f\in H^1(Q_T)$ *bo'lsin, (1)-(3) masalalarda* $\varphi\in H_D^2(D)$, $\psi\in H_D^1(D)$ *va, (1), (2), (4) masalalarda bo'lsa* $\varphi\in H_N^2(D)$, $\psi\in H_N^1(D)$ *bo'lsin. U holda* $p=0,1,2$ *uchun* p*-marta* t *bo'yicha hadlarida differensiallanuvchi (5) qatordan* $H^{2-p}(D_t)$ *da* $t\in[0,T]$ *bo'yicha tekis yaqinlashuvchi qator olinadi va (5) qatordagi* $u(x,t)$ *yig'indi (1)-(3) yoki (1), (2), (4)*

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masalalarning mos deyarli hamma joydagi yechimi bo'ladi. Bundan barcha $t \in [0,T]$ *uchun* $s=2$ *da (8) tengsizlik o'rinli.*

3-teorema. *Aytaylik* $\partial D \in C^{\left[\frac{n}{2}\right]+3}$ *bo'lsin, va (1)-(3) masalalarda* $\varphi \in H_D^{\left[\frac{n}{2}\right]+3}(D)$, $\psi \in H_D^{\left[\frac{n}{2}\right]+2}(D)$, $f \in \tilde{H}_D^{\left[\frac{n}{2}\right]+2}(Q_T)$ *va (1), (2), (4) masalalarda bo'lsa* $\varphi \in H_N^{\left[\frac{n}{2}\right]+3}(D)$, $\psi \in H_N^{\left[\frac{n}{2}\right]+2}(D)$, $f \in \tilde{H}_N^{\left[\frac{n}{2}\right]+2}(Q_T)$ *bo'lsin. U holda (5) qator* $C^2(\bar{Q}_T)$ *da yaqinlashuvchi va uning* $u(x,t)$ *yig'indisi mos qoyilgan masalaning klassik yechimi bo'ladi. Bunda quyidagi tengsizlik o'rinli*

$$\|u\|_{C^p(\bar{Q}_T)} \le C\left(\|\varphi\|_{H^{\left[\frac{n}{2}\right]+p+1}(D)} + \|\psi\|_{H^{\left[\frac{n}{2}\right]+p}(D)} + \|f\|_{H^{\left[\frac{n}{2}\right]+p}(Q_T)}\right), \quad p=0,1,2. \qquad (10)$$

Sobolev sinfida juft tartibli elliptik tenglamalar uchun chegaraviy masalaning umumlashgan yechimining silliqligi.

$G \subset \mathrm{R}^n$ $n-$o'lchovli chegaralangan sohada

$$Lu = (-\Delta)^m u + a(x)u = f(x),\ x \in G \qquad (11)$$

juft tartibli elliptik tenglama berilgan, bunda koeffitsientlari haqiyqiy qiymatga ega va barcha $x \in G$ uchun $a(x) \in C(\bar{G}), a(x) \ge 0$ shartini qanoatlantiradi. Umuman olganda tenglamaning $u(x)$ yechimi va $f(x)$ ozod hadi kompleks qiymatga ega bo'lishi mumkin.

Agar $C^{2m}(G)\bigcap C^{\left[\frac{2m-1}{2}\right]}(\bar{G})$ sinfdagi $u(x)$ funksiya G da (11) tenglamani va ∂G chegarasida

$$u\big|_{x\in\partial G} = 0, \frac{\partial u}{\partial n}\bigg|_{x\in\partial G} = 0, \ldots, \frac{\partial^{\left[\frac{2m-1}{2}\right]} u}{\partial n^{\left[\frac{2m-1}{2}\right]}}\Bigg|_{x\in\partial G} = 0. \qquad (12)$$

shartlarini qanaotlantirsa, u holda bu $u(x)$ funksiya (11) tenglama uchun Dirixle masalasining klassik yechimi deyiladi. Agar $C^{2m}(G)\bigcap C^{2m-2}(\bar{G})$ sinfdagi $u(x)$ funksiya G da (11) tenglamani va ∂G chegarasida

$$u\big|_{x\in\partial G} = 0, \frac{\partial u}{\partial n}\bigg|_{x\in\partial G} = 0, \ldots, \frac{\partial^{\left[\frac{2m-1}{2}\right]} u}{\partial n^{\left[\frac{2m-1}{2}\right]}}\Bigg|_{x\in\partial G} = 0. \qquad (13)$$

chegaraviy shartlarini qanoatlantiruvchi $u(x)$ funksiya G sohada (11) tenglama uchun Rike masalalarining klassik yechimi deyiladi.

$k \geq 1$ uchun $H^k(G)$ fazoning

$$u\big|_{x\in\partial G} = 0, \frac{\partial u}{\partial n}\Big|_{x\in\partial G} = 0, \ldots, \frac{\partial^{\left[\frac{k-1}{2}\right]} u}{\partial n^{\left[\frac{k-1}{2}\right]}}\Bigg|_{x\in\partial G} = 0. \qquad (14)$$

chegaraviy shartlarni qanoatlantiruvchi barcha f funksaiyalaridan tuzilgan qismfazosini $\overset{0}{H}{}^k(G)$ deb belgilaymiz.

$k \geq 1$ uchun $H^k(G)$ fazoning

$$u\big|_{x\in\partial G} = 0, \Delta u\big|_{x\in\partial G} = 0, \ldots, \Delta^{\left[\frac{k-1}{2}\right]} u\Big|_{x\in\partial G} = 0. \qquad (15)$$

chegaraviy shartlarni qanoatlantiruvchi barcha f funksaiyalaridan tuzilgan qismfazosini $H_D^k(G)$ deb belgilaymiz.

$f \in L_2(G)$ bo‘lsin. $\overset{0}{H}{}^m(G)$ fazoga tegishli va barcha $v \in \overset{0}{H}{}^m(G)$ da

$$\int_G \sum_{m_1+\ldots+m_n=m} \frac{m!}{m_1! \cdot \ldots \cdot m_n!} \frac{\partial^m u}{\partial x_1^{m_1} \cdot \ldots \cdot \partial x_n^{m_n}} \frac{\partial^m \overline{v}}{\partial x_1^{m_1} \cdot \ldots \cdot \partial x_n^{m_n}} dx_1 \ldots dx_n = $$
$$= \int_G f\,\overline{v}\,dx_1 \ldots dx_n \qquad (16)$$

integral aniyatini qanoatlantiruvchi u funksiyani (11), (12) Dirixle masalasining umumlashgan yechimi deb ataymiz. $H_D^m(G)$ ga tegishli, barcha $v \in H_D^m(G)$ da (3.3.16) integral aniyatini qanoatlantiruvchi u funksiyasini (11), (13) Rike masalasining umumlashgan yechimi deb ataymiz.

Quyidagi teorema o‘rinli.

4-teorema. $\partial G \in C^m$ *bo‘lsin. Agar* $C^{2m}(\overline{G})$ *sinfga tegishli* $u(x)$ *(11), (12) masalaning (mos ravishda (11), (13) masalaning) klassik yechimi bo‘lsa, u holda bu* $u(x)$ *funksiya ushbu masalalarning umumlashgan yechimi bo‘ladi.*

(12) shartli va (mos ravishda (13) shartli)

$$Lu = (-\Delta)^m u + a(x)u,\ x \in G \qquad (17)$$

operator uchun umumlashgan xos funksiyalarini aniqlash mumkin.

5-teorema. $\partial G \in C^m$ *bo‘lsin. Agar* $u(x) \in H^s$ *bo‘lib,* $s > 2m + \frac{n}{2}$ *bo‘lganida (12) shartli (mos ravishda (13) shartli)*

$$Lu = \left(-\Delta\right)^m u + a(x)u\,,\ x \in G \tag{18}$$

operator uchun umumlashgan xos funksiyasi klassik xos funksiya bo‘ladi.

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