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SILLIQ KO‘PXILLIKLARNING BA’ZI TATBIQLARI

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Annotatsiya: *Geometriyaning jadal rivojlanayotgan va matematikaning boshqa bo’limlarida, umuman barcha sohalarda keng amaliy tadbiqqa ega bo’limlaridan biri hisoblangan silliq ko‘pxillik haqida so‘z boradi. Uning yordamida bir yordamida geometrikning yechimlari ko‘rsatilgan.*
Kalit so‘zlar*: ko‘pxilliklar, silliq ko’pxilliklar, differensiallanuvchi ko‘pxillik.*

Аннотация: *Речь идет о гладком полиноме, который считается одним из быстро развивающихся и широко применяемых разделов геометрии в других разделах математики, в общем, во всех областях. С ее помощью показываются решения геометрического.*
Key words: *polynomials, smooth polynomials, differentiable polynomials.*

Abstract: *It is about the smooth polynomial, which is considered one of the rapidly developing and widely applied branches of geometry in other branches of mathematics, in general, in all fields. With its help, the solutions of the geometric one are shown.*
Ключевые слова: *полиномы, гладкие полиномы, дифференцируемые полиномы.*

Silliq ko‘pxilliklar qatlamalar nazariyasi fanining bir qismi bo‘lib, bu fan o’tgan asrning ikkinchi yarimlarida differensial tenglamalar va diffirensial topologiyaning kesishmasida paydo bo’lgan nazariya hisoblanadi.
Bu maqolada differensiallanuvchi silliq ko‘pxilliklarni aniqlash hamada ular yordamida masalalar yechishning qulay usullari keltirilgan. Bu esa talabalarni geometrik masalalarni yechishda muhim ahamiyat kasb etadi.

Bizga sanoqli bazaga ega bo’lgan M topologik fazo berilgan bo’lsin. M ning $\forall p \in M$ nuqtasi R^n ning biror ochiq qism to’plami $D \subset R^n$ ga gomeomorf $\exists U_p$ atrofga ega bo’lsa, M topologik fazo n o’lchamli topologik ko’pxillik deyiladi. M ko’pxillikning o’lchami dimM belgilanadi: dimM=n.

Misol-1.

a) □ – natural sonlar to’mlami necha o’lchamli ko’pxillik?

b) □ – ratsional sonlar to’plami necha o’lchamli ko’pxillik?

c) I – barcha irratsional sonlar to’plami necha o’lchamli ko’pxillik?

Yechish:

a) $\forall n \in N, O_\varepsilon(n) = U_n$ (p nuqtaning o’rniga n oldik). $D \subset □^1$ topilib, $U_n □ D$ gomeomorf bo’lishini tekshirish kerak.Bular gomeomorf bo’lmaydi chunki, $□^1$ da.

D ochiq intervallar bo'lib, o'lchami sanoqsiz. U_n ning esa o'lchami sanoqli. Demak $U_n \cong D$ emas. Ta'rifga ko'ra $\mathbb{N}$ – natural sonlar to'plami ko'pxillik emas.

b) $\mathbb{Q}$ – ratsional sonlar to'plamida ham gomeomorf sharti bajarilmaydi,chunki $\mathbb{Q}$ – ratsional sonlar to'plami sanoqli to'plam, D ochiq intervallar bo'lib, o'lchami sanoqsiz. Demak ta'rifga ko'ra $\mathbb{Q}$ - ko'pxillik emas.

c) I – barcha irratsional sonlar to'plami o'lchami sanoqsiz, D ham ochiq intervallar bo'lib, o'lchami sanoqsiz. Biroq I – barcha irratsional sonlar to'plami ochiq atrofga ega emas, ya'ni U_p atrof ochiq emas-Int$I$=0, bundan ko'rinadiki I – barcha irratsional sonlar to'plami ko'pxillik emas.

Misol-2. To'g'ri chiziq R^1 bir o'lchamli topologik ko'pxillik.

Haqiqatan ham, $M=(a,b)$ baza bo'lib, $\forall p \in (a,b)$ olamiz, $\exists \varepsilon > 0$ uchun $0_\varepsilon(p) \subset (a,b) \Rightarrow id: 0_\varepsilon(p) \to \mathbb{R}^1$ ya'ni p nuqtani o'zini o'ziga akslantirish bo'ladi. $\mathbb{R}^1$ fazoda $id(x) = x \in \mathbb{R}^1$. Bundan ko'rinadiki, to'g'ri chiziq R^1 bir o'lchamli topologik ko'pxillikdir.

O'lchami R^{n+1} ga teng bo'lgan Evklid fazosining $(n+1)$-o'lchamli $\{(x_1, x_2..., x_{n+1});\ x_i \in R\ (i=1,2,..,n+1)\ x_1^2 + x_2^2 + ... + x^2_{n+1} = 1\}$ qism to'plamini S^n orqali belgilaymiz va n-o'lchamli sfera deymiz. So'ngra, n-o'lchamli Evklid R^n fazoning$\{(x_1, x_2,..., x_n);\ x_i \in R\ (i=1,2,..,n+1)\ x_1^2 + x_2^2 + ... + x_n^2 \le 1\}$ qism to'plamini esa D^n orqali belgilaymiz va n-o'lchamli shar deymiz. Shu ta'riflardan kelib chiqqan holda, $S^{n-1} \subset D^n$ sharning qism to'plami bo'ladi.

$D^n - S^{n-1}$ to'plamni D^n sharning ochiq qism to'plami deb hisoblaymiz va uni $IntD^n$ kabi belgilaymiz.

R^{n+1} dagi standart koordinatalardan foydalangan holda S^n dagi nuqtalar $(n+1)$ ta sonlar sistemasi orqali quyidagicha $x_1, x_2,..., x_{n+1}$ ifoda etiladi. Shu nuqtalar quyidagi munosabatga bo'ysunadi: $x_1^2 + x_2^2 + ... + x^2_{n+1} = 1$.

S^n dan quyidagi qism to'plamlarni ajratib olamiz:

$$U_i^+ = \{(x_1, x_2,..., x_{n+1}); x_1^2 + x_2^2 + ... + x_{n+1}^2 = 1,\ x_i > 0\},$$

$$U_i^- = \{(x_1, x_2,..., x_{n+1}); x_1^2 + x_2^2 + ... + x_{n+1}^2 = 1,\ x_i < 0\}$$

Agar $(x_1, x_2,...., x_{n+1})$ nuqtalar U_i^+ to'plamda berilgan bo'lsa, $x_1, x_2,..., x_{i-1}, x_{i+1},...x_{n+1}$ haqiqiy sonlar bir biriga o'zaro bog'liq bo'lmagan holda o'zgaradi va ularning soni n ga teng bo'ladi. Agar U_i^+ to'plamdagi $(x_1, x_2,...., x_{n+1})$

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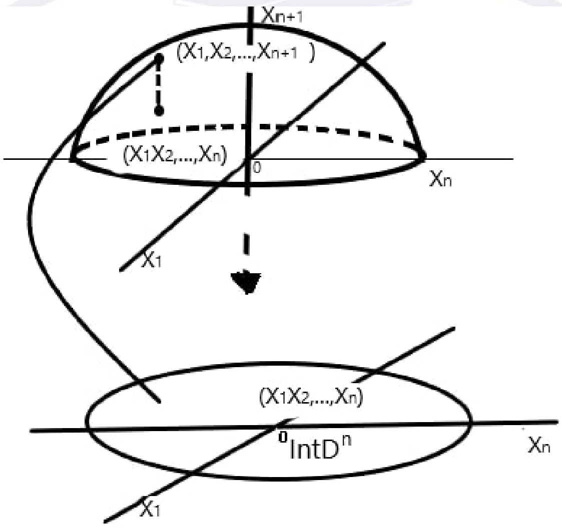
nuqtalarning o‘rniga $IntD^n$ dagi $(x_1, x_2, ..., x_{i-1}, x_{i+1}, ... x_{n+1})$ nuqtalarni mos ravishda qo‘yib chiqsak, quyidagi akslantirish hosil bo‘ladi:

$$\varphi_i^+ : U_i^+ \to IntD^n$$

va bu akslantirishimiz gomeomorfizm bo‘ladi, bu yerda $i = n+1$.



1-chizma

$IntD^n$ dagi nuqtalar n ta o‘zaro bog’liq bo‘lmagan $x_1, x_2, ..., x_n$ sonlar orqali beriladi va ularni $IntD^n$ ning koordinatalari deb qarasak bo‘ladi. S^n sferada hamma (ixtiyoriy) sferalarga mos keluvchi koordinata kiritib bo‘lmaydi. S^n sferadagi U_i^+ qism to‘plamni n ta o‘zaro bog’liq bo‘lmagan haqiqiy sonlar – koordinatalar yordamida $\varphi_i^+ \left(yoki \left(\varphi_i^+\right)^{-1} \right)$ akslantirish orqali ko‘rsatiladi.

S^n dagi bitta nuqta yuqoridagi mulohazalardan keyin har xil koordinatalar orqali ifodalanadi va ular orasidagi munosabatni qaraymiz:

$U_1^+ \cap U_2^+ \ni (x_1, x_2, ..., x_{n+1}),\ x_1 > 0\ va\ x_2 > 0$ berilgan bo‘lsa, unda

$$\varphi_1^+ (x_1, x_2, ..., x_{n+1}) = (x_2, x_3, ..., x_{n+1})$$

$$\varphi_2^+ (x_1, x_2, ..., x_{n+1}) = (x_1, x_3, ..., x_{n+1})$$

Lekin bu holatda kompazitsiya $\varphi_2^+ \circ \left(\varphi_1^+\right)^{-1} : \varphi_1^+ \left(U_1^+ \cap U_2^+\right) \to \varphi_2^+ \left(U_1^+ \cap U_2^+\right)$ va $(y_1, y_2, ... y_n) \in IntD^n$ uchun quyidagi tenglik orqali ko‘rsatiladi:

$$\varphi_2^+ \circ \left(\varphi_1^+\right)^{-1} (y_1, y_2, ..., y_n) = \left(\sqrt{1 - \sum_{i=1}^{n} y_i^2}, y_2, ..., y_n \right),$$

va u gomeomorfizm bo‘ladi. S^n sfera U_i^+ va U_i^- $(i = 1, ... n+1)$ to‘plamlarning $2n+2$ birlashmasi bo‘lib, S^n sferada ixtiyoriy lokal φ_i^+, φ_i^- koordinatalarini kiritish

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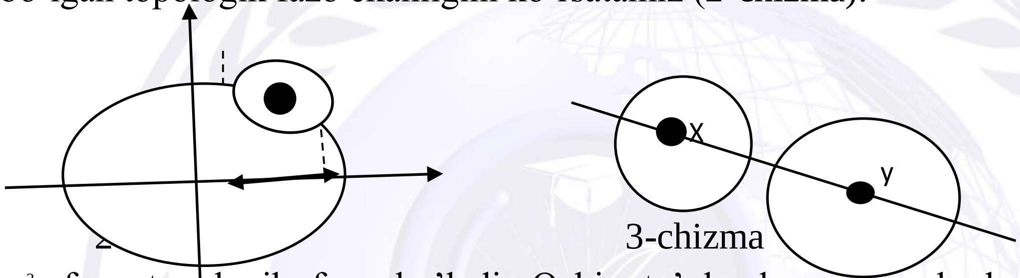
mumkin, bunda $\varphi_2^+ \circ \left(\varphi_1^+\right)^{-1}$ akslantirish “ixtiyoriy marta differensiallanuvchi funksiya” bo‘ladi.

Misol-3. $M = \left\{(x;y) \in R^2 : \frac{x^2}{4} + \frac{y^2}{1} = 4\right\}$ to’plamning silliq ko’pxillik ekanligini isbotlang.

Demak bu ko’pxilligimiz ellips bo’lib, unda $a = 4$ $b = 2$ larga teng . 1) Yuqorida keltirilgan silliq ko’pxillik ta’rifiga asosan ellipsni sanoqli bazaga ega bo’lgan topologik fazo ekanligini ko’rsatamiz (2-chizma):



3-chizma

R^2 fazo topologik fazo bo’ladi. Ochiq to’plamlar ochiq sharlar industrlangan topologiya bilan ellipsni topologik fazo ekanligini ko’rsatamiz. R^2 ni sanoqli sondagi ochiq sharlar bilan qoplash mumkin. R^2 sanoqli bazaga ega. Industrlab hosil qilingan topologiya yordamida ellips topologik fazo hosil qiladi. U ham sanoqli bazaga ega. Xausdorf fazo bo’ladi: $\varepsilon < \frac{xy}{2}$ deb olsak (3-chizma).

2) Ta’rifga asosan C^r – moslashganlikka tekshiramiz.

$U_1^+ = \{(x;y) \in M : x > 0\}$ $\qquad \varphi_1^+ : U_1^+ \to (-2;2)$

$U_1^- = \{(x;y) \in M : x < 0\}$ $\qquad \varphi_1^- : U_1^- \to (-2;2)$

$U_2^+ = \{(x;y) \in M : y > 0\}$ $\qquad \varphi_2^+ : U_2^+ \to (-4;4)$

$U_2^- = \{(x;y) \in M : y < 0\}$ $\qquad \varphi_2^- : U_2^- \to (-4;4)$

$A = \left\{\left(U_1^+, \varphi_1^+\right), \left(U_1^-, \varphi_1^-\right), \left(U_2^+, \varphi_2^+\right), \left(U_2^-, \varphi_2^-\right)\right\}$ atlas bo’ladi. Kartalarni C^r – moslashganlikka tekshirsak quyidagicha bo’ladi.

a) $\left.\begin{array}{l} U_1^+ \cap U_1^- = \varnothing \\ U_2^+ \cap U_2^- = \varnothing \end{array}\right\}$ C^r – moslashgan.

b) $\left.\begin{array}{l} U_1^+ \cap U_2^+ \neq \varnothing \\ U_1^- \cap U_2^- \neq \varnothing \end{array}\right\}$ C^r – moslashganni birinchi shartini qanoatlantirmaydi, demak ikkinchi shart – “agar $U \cap V \neq \emptyset$ bo’lganda $\psi^\circ\varphi$ diffiomorfizm mavjud bo’lsa, C^r- moslashgan bo’ladi” ga asosan diffiomorfizmga tekshiramiz.

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$$\varphi_1^+\circ(\varphi_2^+)^{-1}:\varphi_2^+(U_1^+\cap U_2^+)\to\varphi_1^+(U_1^+\cap U_2^+)\qquad \varphi_2^+\circ(\varphi_1^+)^{-1}:\varphi_1^+(U_1^+\cap U_2^+)\to\varphi_2^+(U_1^+\cap U_2^+)$$

$$\varphi_1^+\circ(\varphi_2^+)^{-1}(x)=2\sqrt{1-\frac{x^2}{16}},\qquad \varphi_2^+\circ(\varphi_1^+)^{-1}(y)=4\sqrt{1-\frac{x^2}{4}}.$$

$$\varphi_1^-\circ(\varphi_2^-)^{-1}:\varphi_2^-(U_1^-\cap U_2^-)\to\varphi_1^-(U_1^-\cap U_2^-)\qquad \varphi_2^-\circ(\varphi_1^-)^{-1}:\varphi_1^-(U_1^-\cap U_2^-)\to\varphi_2^-(U_1^-\cap U_2^-)$$

$$\varphi_1^-\circ(\varphi_2^-)^{-1}(x)=-2\sqrt{1-\frac{x^2}{16}},\qquad \varphi_2^-\circ(\varphi_1^-)^{-1}(y)=-4\sqrt{1-\frac{x^2}{4}},$$

Bular diffiomorfizm va bundan kartalar C^r- moslashgan. Ta’rifga ko’ra ellips silliq ko’pxillik ekan.

Foydalanilgan adabiyotlar.

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